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Free vibrations of interspersed railway track systems in three-dimensional space

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Abstract

Statistically, the actual loading conditions for railway tracks are rather dynamic and transient. The dynamic loadings due to train and track interactions redistribute from the rails to the rail pad, from the rail pad to the railway sleeper, and from the railway sleeper to the underlying ground. Commonly, railway sleeper in track systems is modeled as a beam on elastic foundation. This study makes use of a calibrated finite element model of railway sleepers in a track system, in order to investigate the resonant frequencies and associated mode shapes of railway components in interspersed track systems. The numerical model takes into account the tensionless characteristic of the elastic support as well as the more realistic partial support condition. Using a finite element package STRAND7, the dynamic finite element model of the railway concrete sleeper was precisely established. The dynamic model has then been extended to demonstrate free vibration behaviours of the railway tracks. The effect of interspersed patterns (1 in 2; 1 in 3; and 1 in 4) on railway track dynamics has been firstly investigated and presented herein.

Keywords

free, three, interspersed, railway, dimensional, vibrations, systems, space, track

Disciplines

Engineering | Science and Technology Studies

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Free vibrations of interspersed railway track systems in three-dimensional space

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Abstract:

Statistically, the actual loading conditions for railway tracks are rather dynamic and transient. The dynamic loadings due to train and track interactions redistribute from the rails to the rail pad, from the rail pad to the railway sleeper, and from the railway sleeper to the underlying ground. Commonly, railway sleeper in track systems is modeled as a beam on elastic foundation. This study makes use of a calibrated finite element model of railway sleepers in a track system, in order to investigate the resonant frequencies and associated mode shapes of railway components in interspersed track systems. The numerical model takes into account the tensionless characteristic of the elastic support as well as the more realistic partial support condition. Using a finite element package STRAND7, the dynamic finite element model of the railway concrete sleeper was precisely established. The dynamic model has then been extended to demonstrate free vibration behaviours of the railway tracks. The effect of interspersed patterns (1 in 2; 1 in 3; and 1 in 4) on railway track dynamics has been firstly investigated and presented herein.

Keywords: railway track systems, interspersed tracks, sleeper/ballast interaction, free vibration analysis, resonance, dynamic mode shapes, tensionless support.

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1. Introduction

A traditional railway track consists of steel rails, sleepers, fasteners, ballast, and formation (capping layer over compacted soil). A review on the loading conditions acting on railway tracks for either passenger or freight trains shows that dynamic behaviour of railway track is vital to understand the track dynamic responses to diverse loading conditions [1]. The critical loading condition, which often causes structural cracks in brittle sleepers, is the large impact loads due to wheel/rail irregularities. A common transient waveform pattern of wheel impacts due to short-pitch rail corrugations can be seen in Fig. 1. Clearly, the magnitude of the impact forces varies from 200kN to 400kN while the duration is ranging from 2 to 10 msec. Using a transient pulse concept, these impact pulses are associated with the vibration excitation frequency range from 100 Hz to 500 Hz ($f = 1/T$: f is a frequency and T is a period). In reality, wheel/rail interaction generates impact forces acting on a rail seat. The pulse load patterns are dependent to train speed, track geometry, axle load, vehicle type, and wheel/rail defects. Rail engineers must take into account the frequency ranges of static and dynamic loadings in design and construction of railway tracks with respect to critical train speeds and operational parameters [1-3].

The effect of ballast conditions on the flexural response of the railway concrete sleepers was established using a finite element model of the railway concrete sleeper [4]. It was found that the static wheel load generally imparts the positive bending moment at the railseat whilst provides the negative bending moment at mid span of the sleepers. The variation of ballast stiffness has a low sensitivity on the flexural responses of the railway concrete sleeper but such variation plays a role in rail responses and track modulus [5-6].

From a quasi-static point of view, the standard design of a railway track is fairly conservative [7]. In contrast, the actual transient loadings excite the track components dynamically and it is up to the capability of such components to filter and redistribute the dynamic force onto adjacent components supporting one another. The dynamic amplification factor is substantially dependent on the ratios between the period of the transient loading and each modal period of the railway track components [8-12]. It is noteworthy that wheel force excitation also applies laterally to track components by virtue of rail cant (to enable conical wheels) and track superelevation (to balance centrifugal effect from the body of trains). In fact, a wheel excitation could resonate track natural frequencies in any particular dynamic modes, which can either induce the corresponding structural damage or emit noises. There are many types of railway noises of which the root causes are contradicting. For example, increasing rail pad stiffness might reduce 'rolling noise', but also has potential to increase 'ground-borne noise' and reduce service lives of adjacent track components [11-12]. Some of the common railway noises due to wheel-rail interactions, community may experience, are [13]:

- rolling noise from wheel-rail roughness interaction
- ground-borne noise from track vibrations
- structural-borne noise from railway bridge vibrations
- wheel flanging noise from the rubbing of wheel flange onto rail in curved tracks
- wheel squeal noise from wheel-rail interactions at resonance.

The frequency range of rolling, ground-borne and structural-borne noises is from 0 to 500 Hz, while the impact noise could vary from a hundred up to 1,000 Hz and often the wheel flanging and squeal noises are found to exceed 4,000 Hz [13].

It is clear that track components play significant role in railway dynamics. In the past (since 1850s), timber was commonly used as railway sleepers in Australia due to the then economy and availability. Timber sleepers tend to have service life about 15-20 years, depending on maintenance and operational situations. Over a period of time, the timber sleepers age and require a major maintenance. In addition, to enhance railway operations, timber sleepers need strengthening. A method, which is a temporary measure, is to partially re-sleeper the track with concrete sleepers. This method is commonly called 'intersperse' [14]. Depending on track stiffness, deterioration process, and operational parameters, there are a variety of interspersing patterns, i.e. 1 in 2; 1 in 3; or 1 in 4 (Note: 1 in 2 means imposing a concrete sleeper every two sleepers). The concrete sleepers used are a generally medium-duty type so the rail levels can be retained. This method has some disadvantages because one only replaces a stiffer concrete sleeper on the aged and soft existing formation, often resulting in a soil foundation failure. Due to track stiffness inconsistency and different decay rates of time-dependent material properties, such a method is normally suitable for a short-term maintenance strategy where track strengthening cannot wait until a major trackwork could be programmed.

This paper presents free vibration behaviours of the interspersed railway tracks in three-dimensional space. To explore the dynamic effects, a variety of interspersing patterns have been established. The finite element model in this study has taken into

account more realistic boundary conditions and load cases. The results provide better insight into the dynamic behaviour of railway track and its components under different interspersed patterns. This paper is aimed at raising the dynamic consideration in the design of ballasted railway tracks. The use of commercial finite element program would enable the industry outreach to researchers and engineering practitioners in order to construct a numerical model to better predict and control the track responses and to develop solutions to vibro-acoustic problems in interspersed railway tracks.

2. Finite element analysis

A two-dimensional Timoshenko beam model was previously developed and found to be one of the most suitable options for modeling concrete sleepers [8-13]. In this study, the finite element models of railway tracks have been developed and calibrated against the numerical and experimental modal parameters [8, 9]. Fig. 2 shows the finite element models in three-dimensional space for an in-situ railway track with different types of sleepers. Using a general-purpose finite element package STRAND7 [15], the numerical model included the beam elements, which take into account shear and flexural deformations, for modeling the sleeper and rails. The 60kg rail cross section and sectional parameters were used in accordance with Australian Standard AS1085.1 [16]. The trapezoidal cross-section was assigned to the concrete sleeper elements in accordance with the standard medium duty sleepers [17-20]. The rectangular cross-section was assigned to the timber sleeper elements in accordance with the standard timber sleepers used in NSW [21]. The rail pads at railseats were simulated using a series of spring-dashpot elements. The distance offset between rails and sleepers was set to 100mm to

more clearly illustrate the track behaviours. This setup does not affect the numerical results [17]. In this study, the stiffness and damping values of HDPE pads were assigned to these springs [22, 23]. The support condition was simulated using the nonlinear tensionless beam support feature in Strand7 [15]. This attribute allows the beam to lift over the support while the tensile supporting stiffness is omitted. The tensionless support option can correctly represent the ballast characteristics in real tracks [15, 17]. It is important to note that the experimental modal testing was first performed to identify structural parameters of the sleepers. Then, the finite element model was developed using available data from the manufacturer. The model was then updated through the comparison of modal parameters. Table 1 shows the geometrical and material properties of the finite element model. Based on previous studies [7-8], effects of length and boundary of track in this study (18 bays or 10.8 m) on the computation and the frequencies of interest are negligible. These data have been validated and the verification results were presented elsewhere [8-10].

According to a literature review, free vibration analyses of interspersed railway track systems have not thoroughly been evaluated. Therefore, numerical simulations are conducted using the nonlinear solver in STRAND7 [15], in order to investigate the effect of interspersing patterns on the natural frequencies and associated dynamic mode shapes of such railway track systems. Note that STRAND7 can take care of both viscous and hysteric damping parameters for dynamic analysis. The study will provide the better understanding into the vibro-acoustic behaviours of interspersed railway tracks before carrying out any design or maintenance process. The understanding will enable rail engineers to make a better decision on asset management and maintenance strategies.

3. Free vibration analyses

Based on the previous analyses [8], the standard design sleepers have been employed to investigate and benchmark the free vibration behaviours between timber and concrete sleepered tracks [20, 21]. It should be noted that the Australian Standard AS1085.14 [2] suggests the use of partial ballast support for a standard gauge sleeper, as shown in Fig. 3. The ballast support underneath the sleeper spans about 1.0 times of the difference between the total length of sleeper and the rail gauge length [24]. For a new rail track construction, the ballast underneath sleeper' central part will not be tamped and sometimes about 50mm void (or more in some practices) between sleeper soffit and ballast will be left in order to minimize the negative bending moment at sleepers' mid-span [25]. This practice allows the realistic ballast pressure distribution to be controlled as per the design process, especially for concrete sleepered tracks [2, 24].

Table 2 and Fig. 4 show the dominant natural frequencies and associated dynamic mode shapes of the timber and concrete sleepered tracks, respectively. It is found from Figs. 4a-c that the lateral dynamic rail bending modes are among the lowest natural frequencies. Body twisting modes can be seen in Figs. 4d-g, whereas these modes may affect dynamic hunting-coupler behaviour of rolling stocks.

Figs. 4h and 4i display the rigid body motions of railway tracks. In these modes, the rail pads (modelled by using a spring-dashpot) enable large-amplitude dynamic axial motions. The sleepers then behave as consistent masses either in-phase or out-of-phase with the rail motions. The results show that a modal phase difference also occurs in the

dynamic bending responses of structural elements. These out-of-phase dynamic mode shapes suppress the acoustic radiation from rail vibrations at such resonant frequencies. It is important to note that the rail pad stiffness plays a key role on the dynamic modes and phase differences. Softer rail pads (e.g. $< 100 \text{ MN/m}$) tend to further diminish such effects.

Figs. 4j-k illustrates the dynamic bending modes of the railway sleepers. Clearly, the maximum positive dynamic bending at rail seats is associated with the second and third bending modes. The negative or hogging bending moment is maximal at the first flexural mode. In reality, it has been commonly found that railway sleepers are often damaged when the load excitations approach these flexural resonant frequencies.

Fig. 5 shows the effect of interspersed pattern 1:2 on the dynamic mode shapes at a higher frequency range. It is clear in Figs. 5 to 7 that the interspersed patterns play a key role in altering dynamic mode shapes of the railway tracks. Crossing-over of natural frequencies at higher frequency range (over 200 Hz) can be observed in the interspersed tracks as tabulated in Table 2. In addition, interspersing a railway timber sleepered track with concrete sleepers is very likely to induce the out of phase vibrations. However, the crossover between mixed dynamic modes between in-phase and out-of-phase flexural modes together with rigid body motions can also be observed, especially with an inclusion of a different element (i.e. an interspersed concrete sleeper) in track systems. This effect is more pronounced for the 1:4 interspersed pattern.

4. Conclusion

Using finite element modeling, this paper investigates the effect of different types of interspersed patterns on the dynamic mode shapes and resonant frequencies of the railway track systems in three-dimensional space. An established dynamic finite element model of railway track was utilized in this study. The free vibration and nonlinear analysis solver in STRAND7 was employed to cope with the tensionless support and the more realistic partial support condition.

At a systems perspective, the free vibration analyses found that the dynamic bending modes are affected significantly by the interspersing practice. The flexural track dynamics can be crossed over in order to magnify alternate interspersed sleepers' vibration amplitudes. It is noticeable that the dynamic flexural amplitudes at mid span of sleepers act out of phase between the first and the second rigid-body resonances. In reality, lowering rigid body resonances can cause more breakage and pulverisation of ballast and formation. Importantly, the interspersed patterns tend to self-initiate the out of phase vibrations that potentially damage the fastening systems of railway tracks. The mixed modes of rigid body motion and out-of-phase vibration can also be observed at a higher frequency range. Note that the partial support condition (see Fig. 3) hardly shows the influence over the dynamic flexural responses at rail seats, but it occasionally alters the dynamic bending moments of sleepers at mid span, by reducing the negative dynamic bending amplitudes at mid span.

Understanding in the free vibration behaviour of railway track systems is vital for structural health monitoring and vibration control strategies as well as rail asset management. This paper demonstrates a finite element modeling for railway tracks as to extend the appropriate methodology to investigate resonant frequencies and associated dynamic mode shapes of interspersed railway track systems. Future work involves the time-dependent analysis of interspersed railway tracks.

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Table 1 Engineering properties of the standard concrete sleeper used in the modeling

Parameter lists		
Flexural rigidity	$EI_c = 4.60, EI_r = 6.41$	MN/m ²
Shear rigidity	$\kappa GA_c = 502, \kappa GA_r = 628$	MN
Ballast stiffness	$k_b = 13$	MN/m ²
Rail pad stiffness: vertical	$k_p = 800$	MN/m
: lateral	$k_p = 400$	MN/m
Sleeper density	$\rho_s = 2,750$	kg/m ³
Sleeper length	$L = 2.5$	m
Rail gauge	$g = 1.5$	M
Sleeper spacing	$s = 0.6$	m

Table 2 Natural frequencies and associated mode shapes of interspersed railway tracks

Dynamic mode shapes	Natural frequencies of different types of railway tracks (Hz)				
	Concrete	1 in 2	1 in 3	1 in 4	Timber
Lateral bending (Fig. 4a)	3.87	4.34	4.58	4.70	5.05
Lateral bending (Fig. 4b)	10.48	11.77	12.42	12.72	13.72
Lateral bending (Fig. 4c)	20.44	22.97	24.23	24.52	26.79
Body twisting (Fig. 4d)	44.69	48.98	51.17	52.25	55.80
Body twisting (Fig. 4e)	46.77	51.38	53.71	54.73	58.69
Body twisting (Fig. 4f)	52.94	58.46	61.16	61.62	62.28
Body twisting (Fig. 4g)	64.85	72.02	75.08	82.46	67.18
Rigid body (Fig. 4h)	112.36			93.28	99.83
Rigid body (Fig. 4i)	114.97	99.73			101.22
Flexural (Fig. 4j)	117.34	101.14; 116.64	101.92; 116.10	104.25; 115.23	101.49
Flexural (Fig. 4k)	267.79	176.22; 248.81	182.81; 224.48		181.47
Flexural (Fig. 4l)	404.32	230.75; 341.59	233.58; 263.24	277.18	260.89
Flexural (Fig. 4m)					440.68
Gauge spreading (Fig. 4n)	307.60				
Out-of-phase (Fig. 5a)		352.20			
Out-of-phase (Fig. 5b)		404.25			
Out-of-phase			193.45 (Fig. 6a)	182.89 (Fig. 7a)	
Out-of-phase			257.16 (Fig. 6b)	235.77 (Fig. 7b)	
Out-of-phase			329.06 (Fig. 6c)	360.62 (Fig. 7c)	
Out-of-phase + Rigid body			398.85 (Fig. 6d)	418.40 (Fig. 7d)	
Out-of-phase + Rigid body				482.14 (Fig. 7e)	

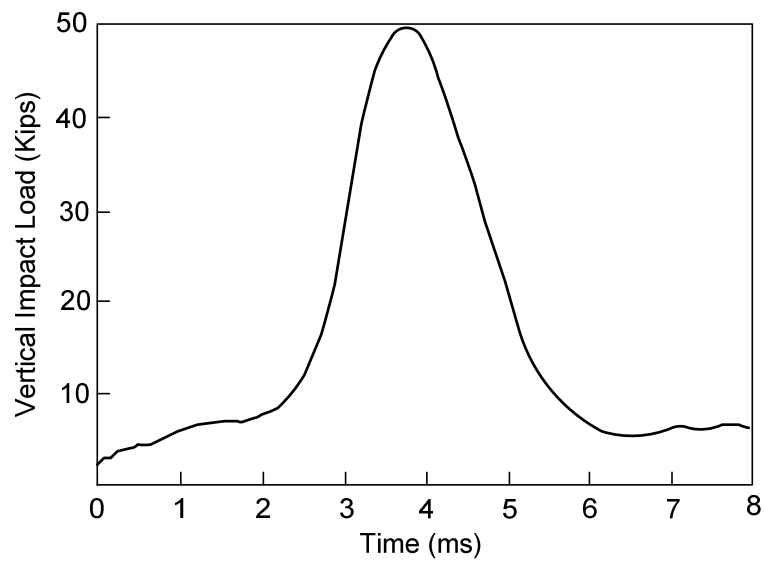
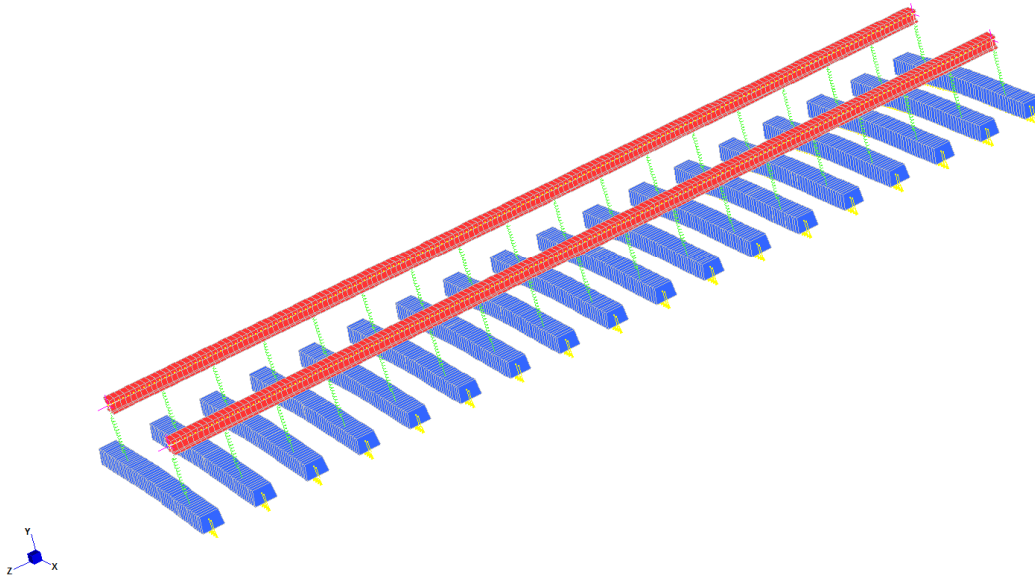
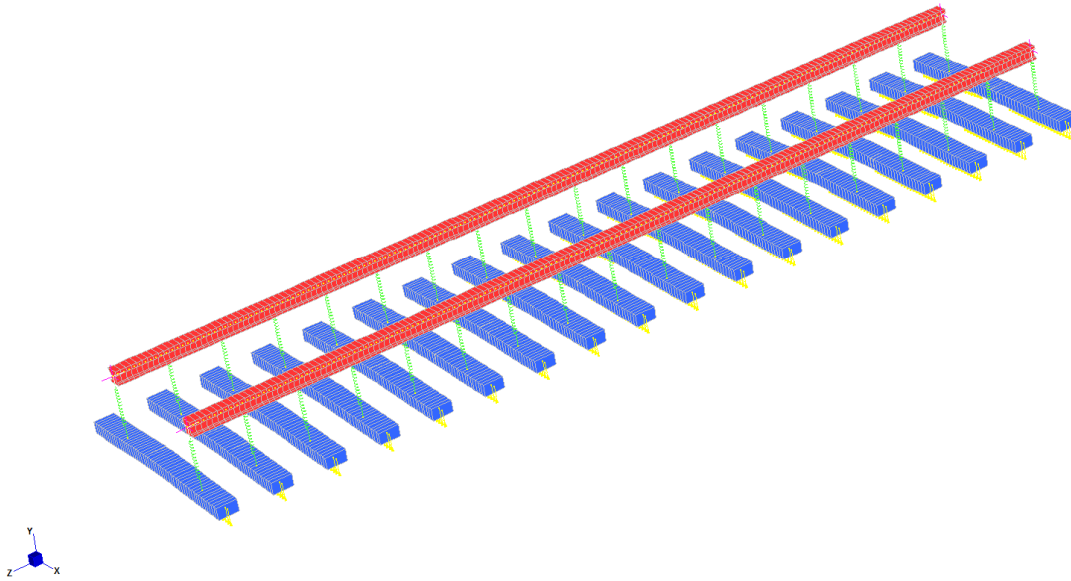


Fig. 1 A typical impact due to a wheel/rail out-of-round defect, 1 Kip = 4.448 kN [1]

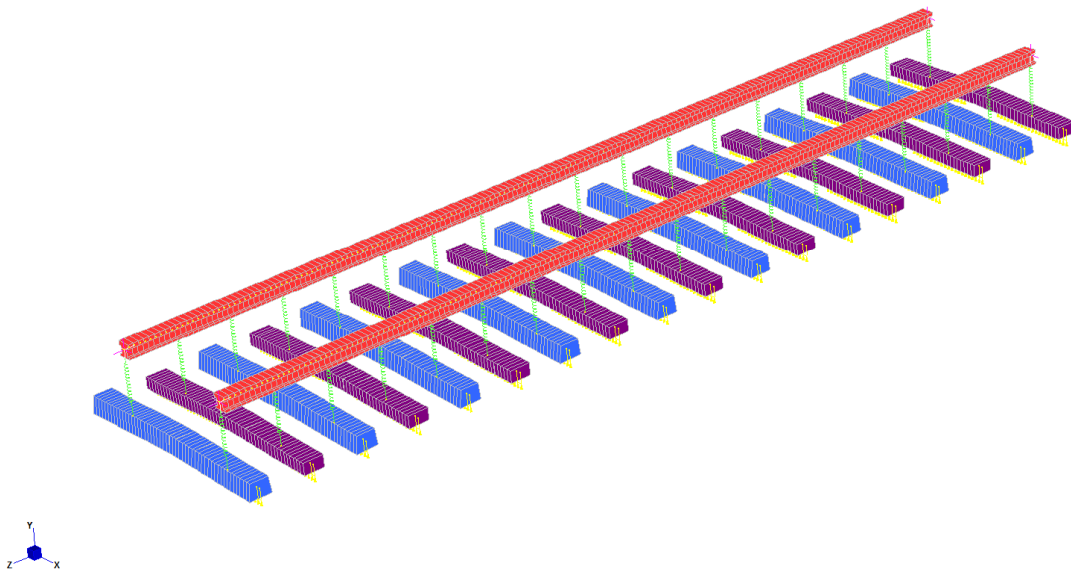


a) concrete sleepered track

Fig. 2. STRAND7 finite element models of railway tracks

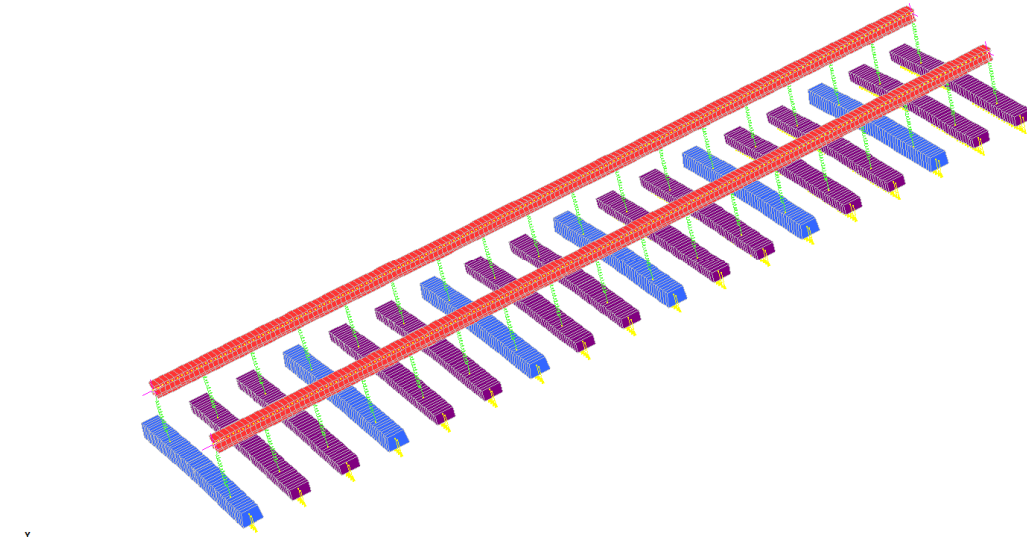


b) timber sleepered track

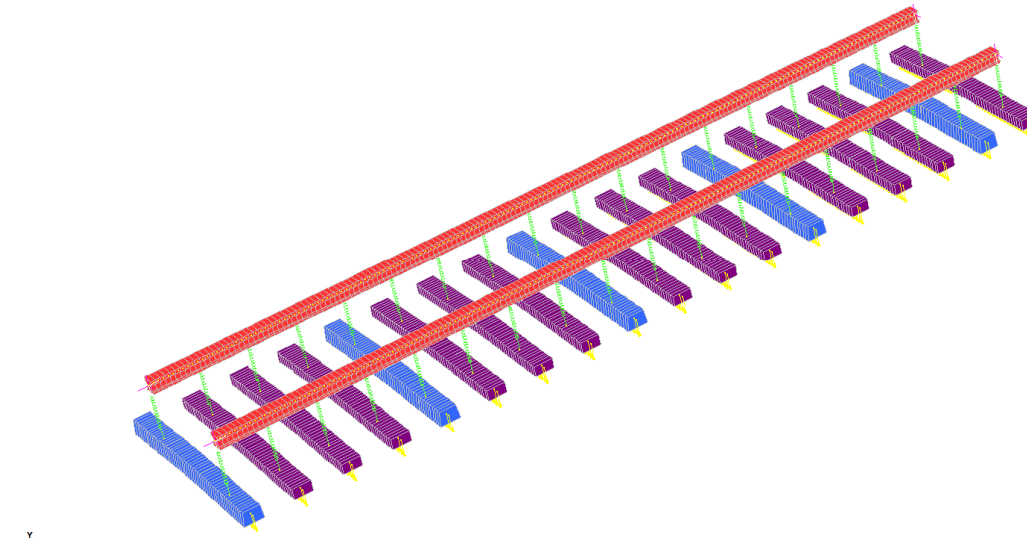


c) 1:2 interspersed track

Fig. 2. STRAND7 finite element models of railway tracks



d) 1:3 interspersed track



e) 1:4 interspersed track

Fig. 2. STRAND7 finite element models of railway tracks

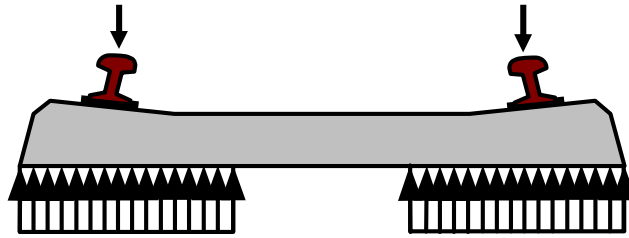
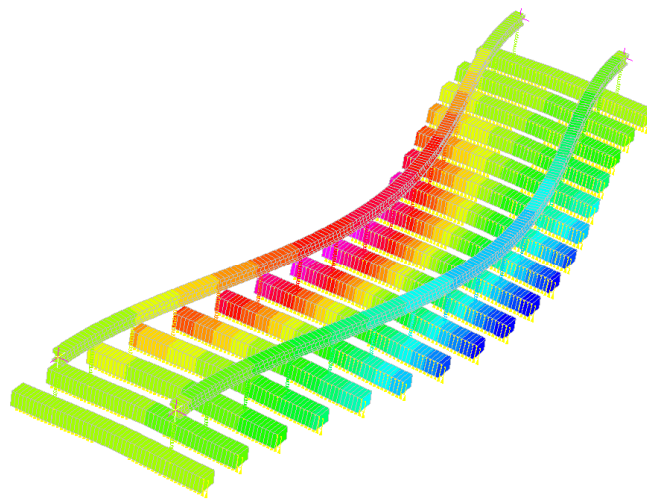
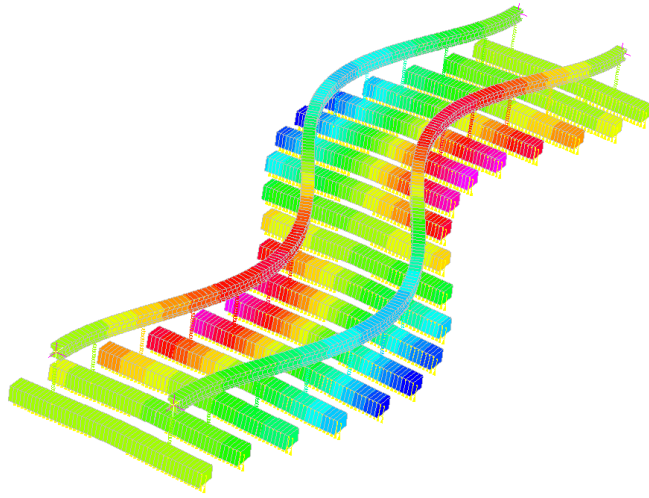


Fig. 3 Standard partial ballast support condition [2, 9-11]

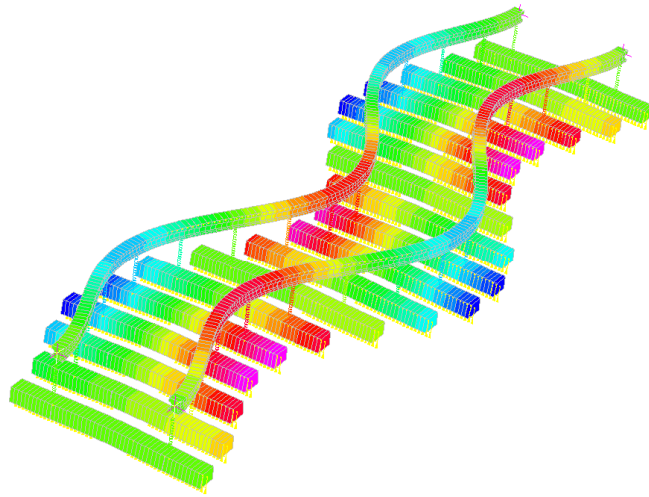


a) lateral motion

Fig.4. Free vibrations of railway tracks (0 to 500 Hz)

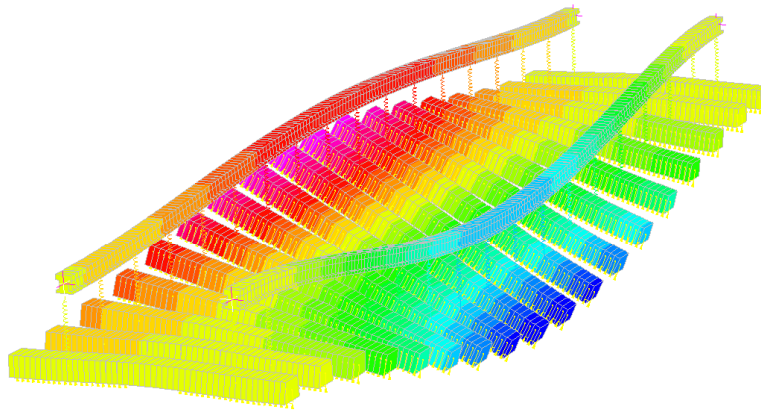


b) lateral motion

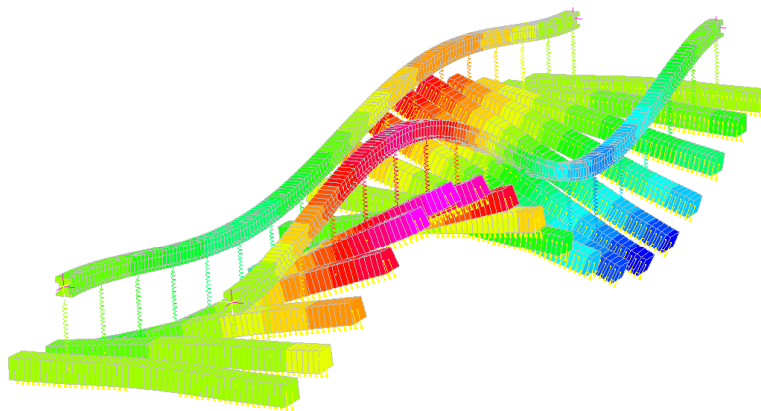


c) lateral motion

Fig.4. Free vibrations of railway tracks (0 to 500 Hz)

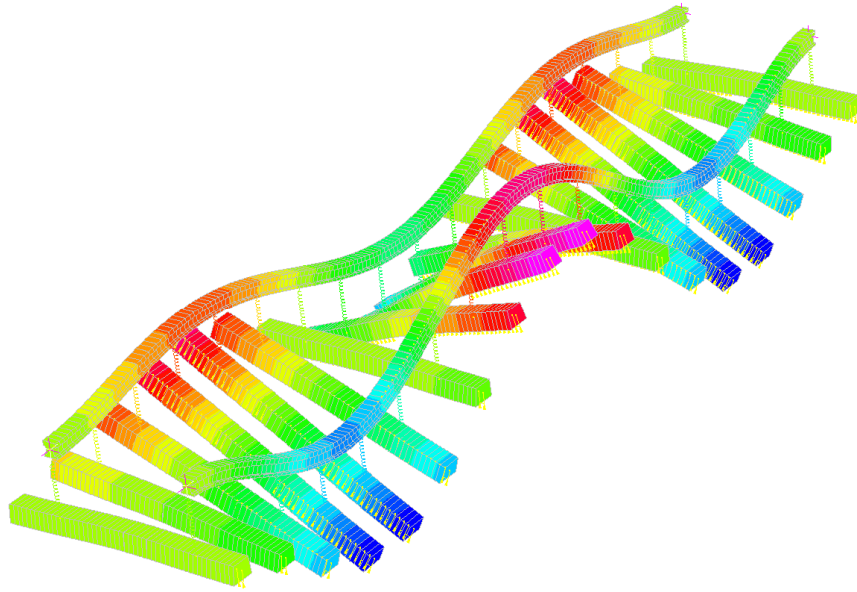


d) rigid body twisting motion

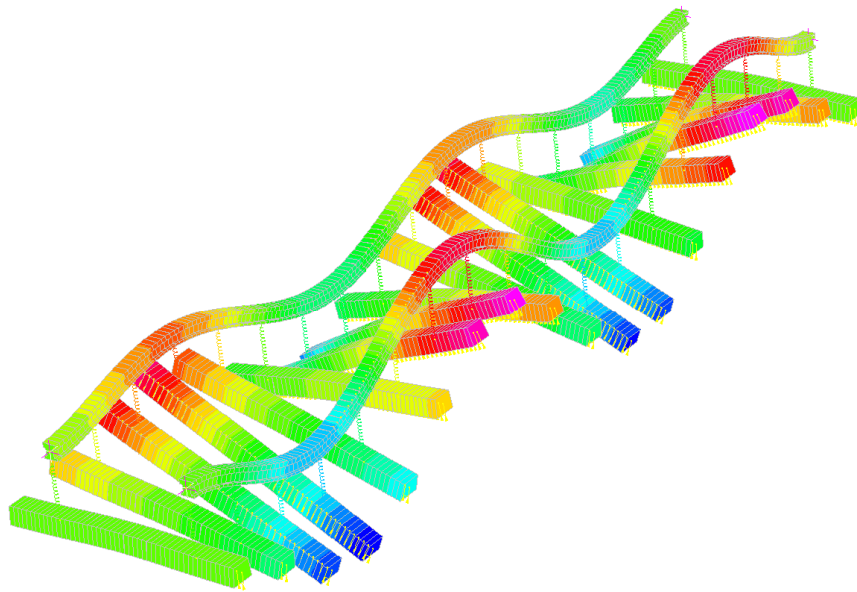


e) rigid body twisting motion

Fig.4. Free vibrations of railway tracks (0 to 500 Hz)

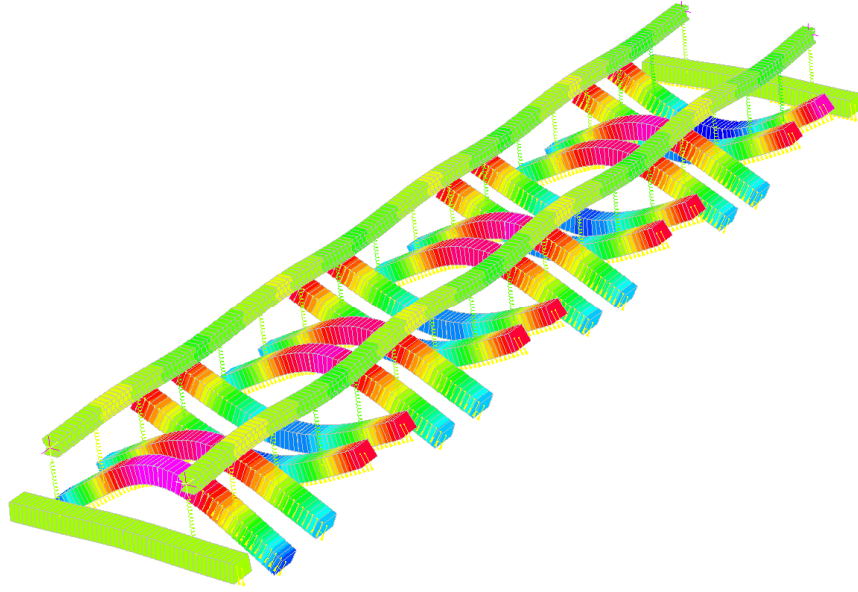


f) rigid body twisting motion

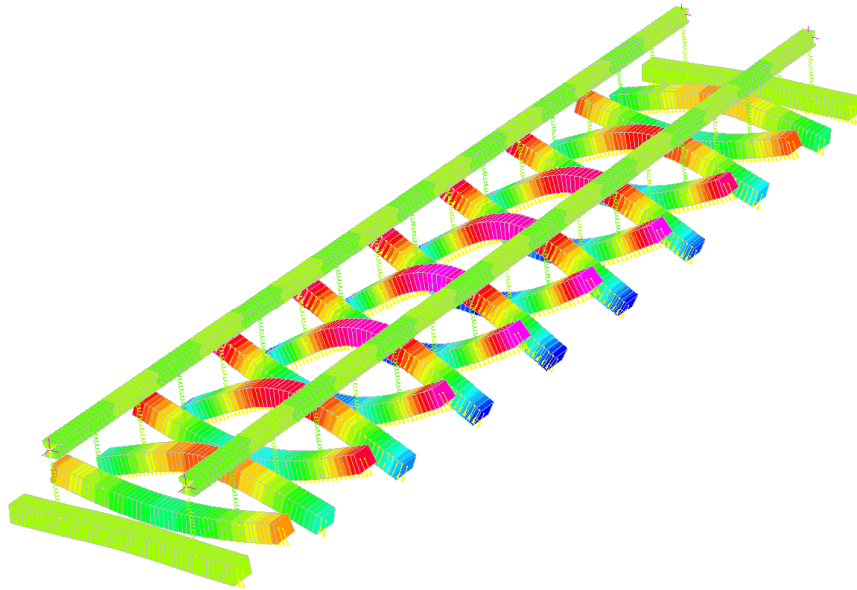


g) rigid body twisting motion

Fig.4. Free vibrations of railway tracks (0 to 500 Hz)

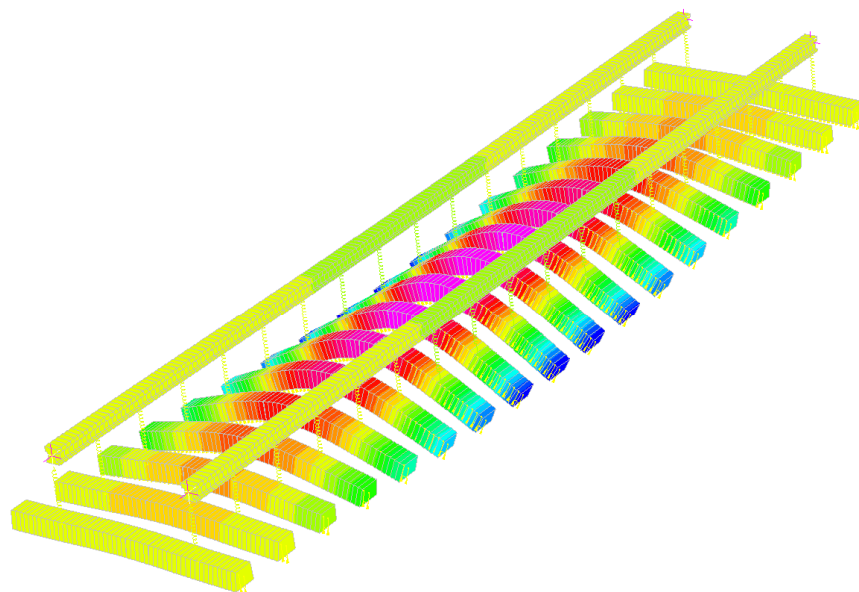


h) rigid body motion

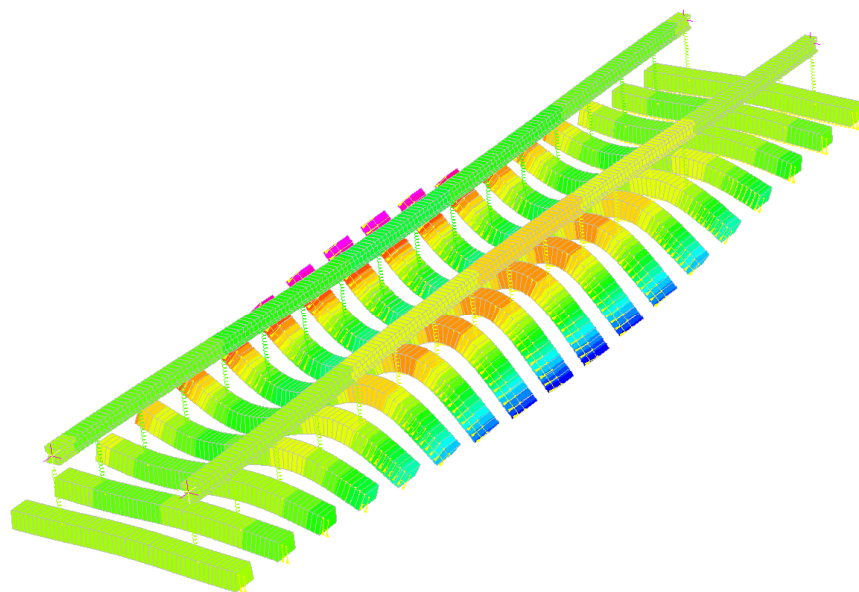


i) rigid body motion

Fig.4. Free vibrations of railway tracks (0 to 500 Hz)

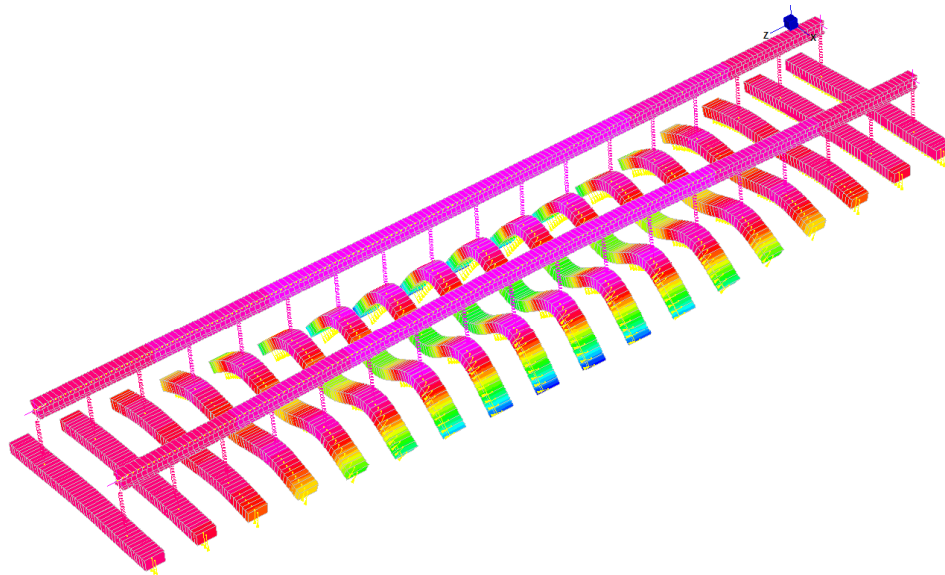


j) flexural mode

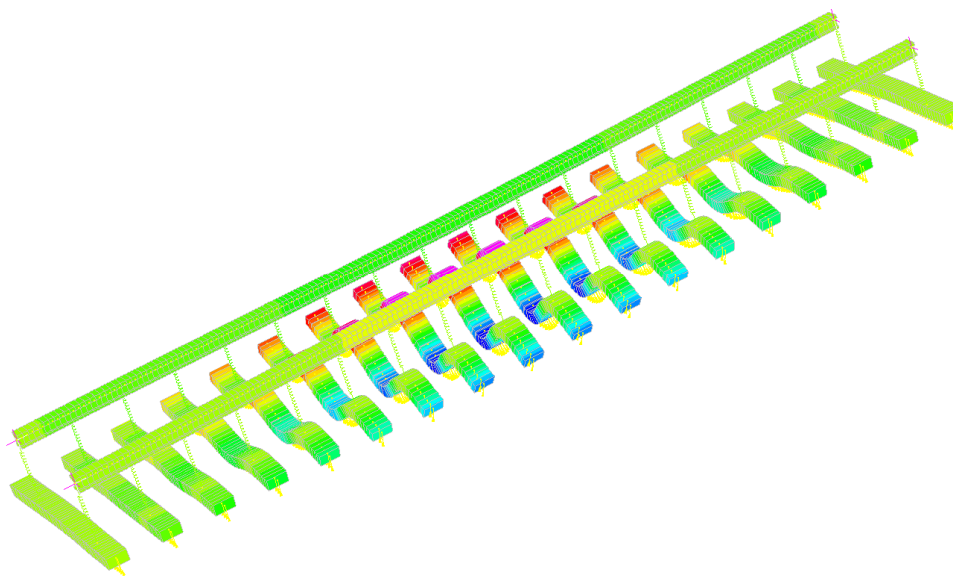


k) flexural mode

Fig.4. Free vibrations of railway tracks (0 to 500 Hz)

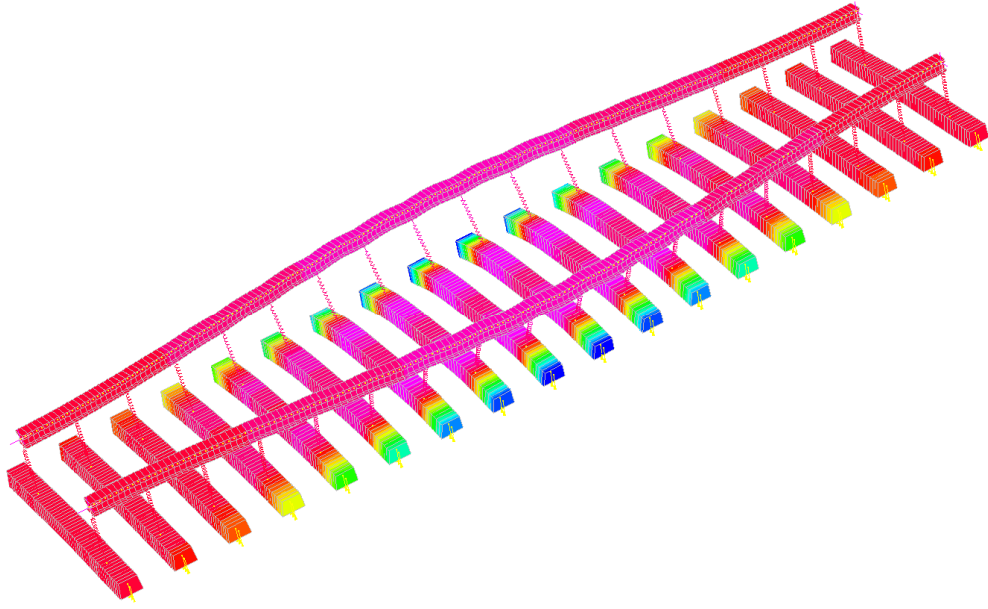


l) flexural mode



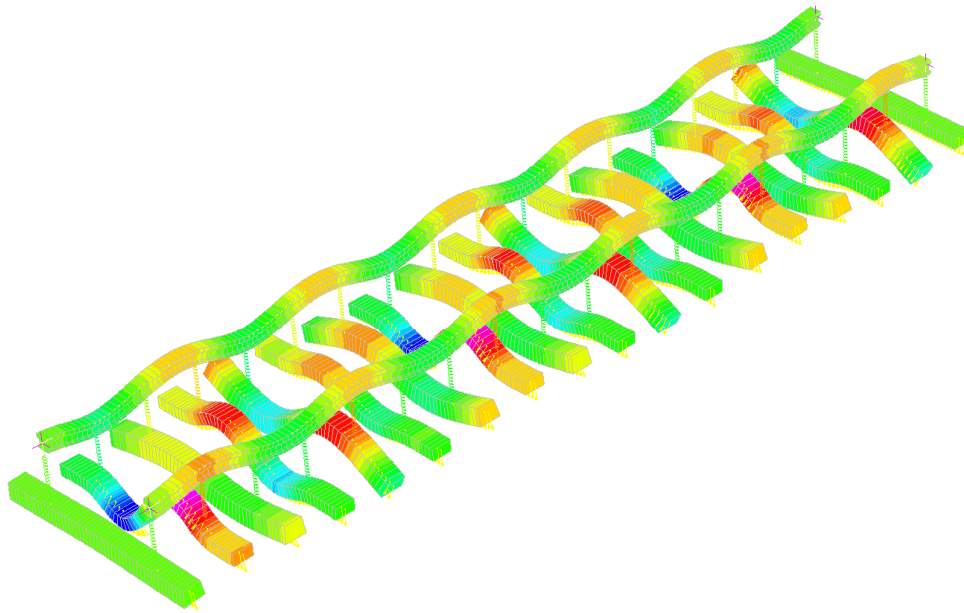
m) flexural mode

Fig.4. Free vibrations of railway tracks (0 to 500 Hz)

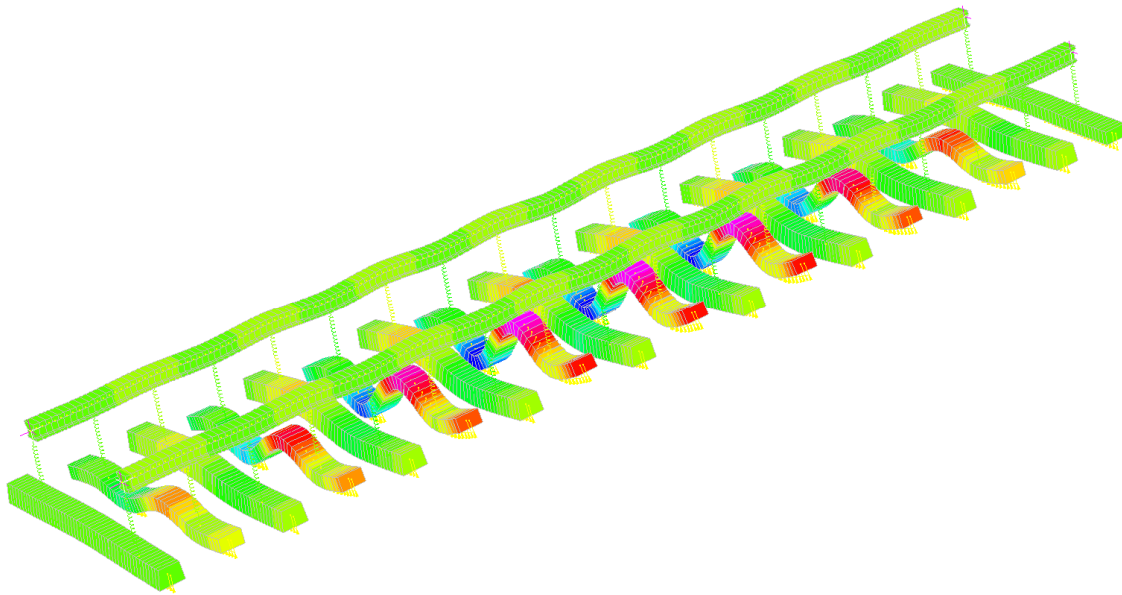


n) gauge spreading mode

Fig.4. Free vibrations of railway tracks (0 to 500 Hz)

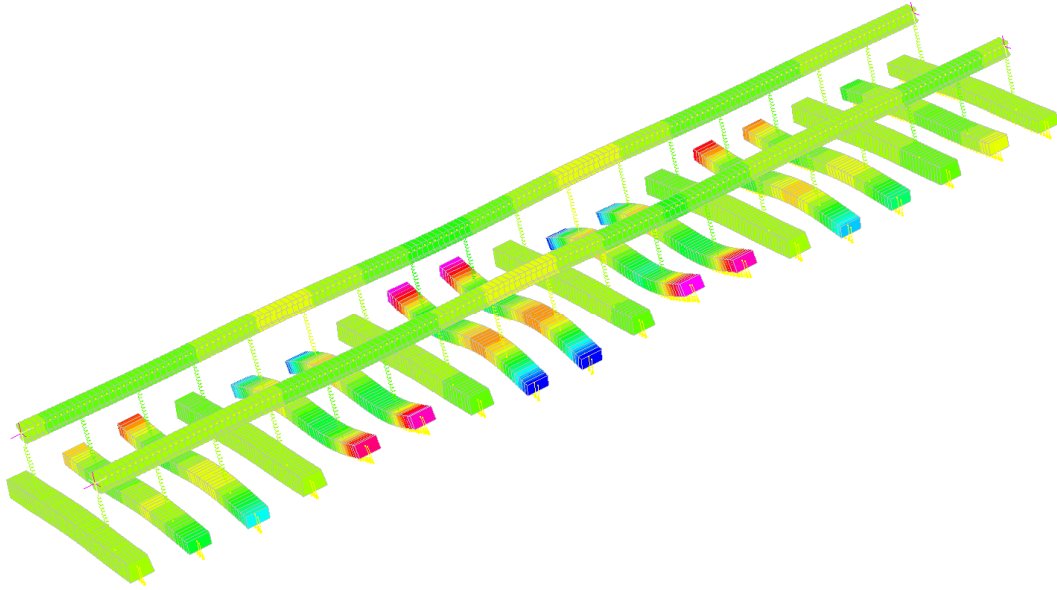


a) out-of-phase bending

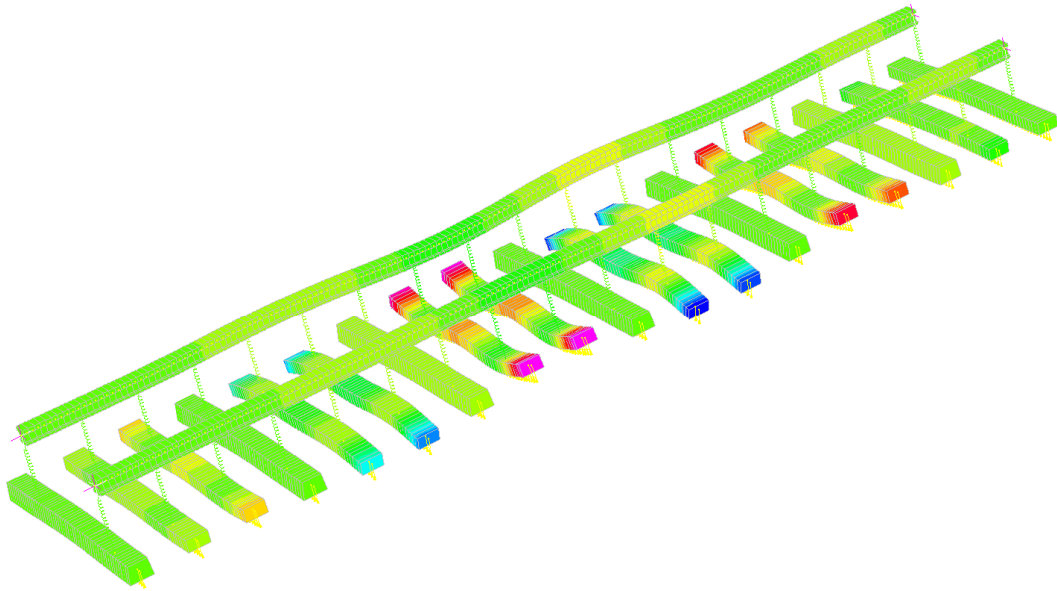


b) out-of-phase bending

Fig. 5. Free vibrations of 1:2 interspersed railway tracks (300 to 500 Hz)

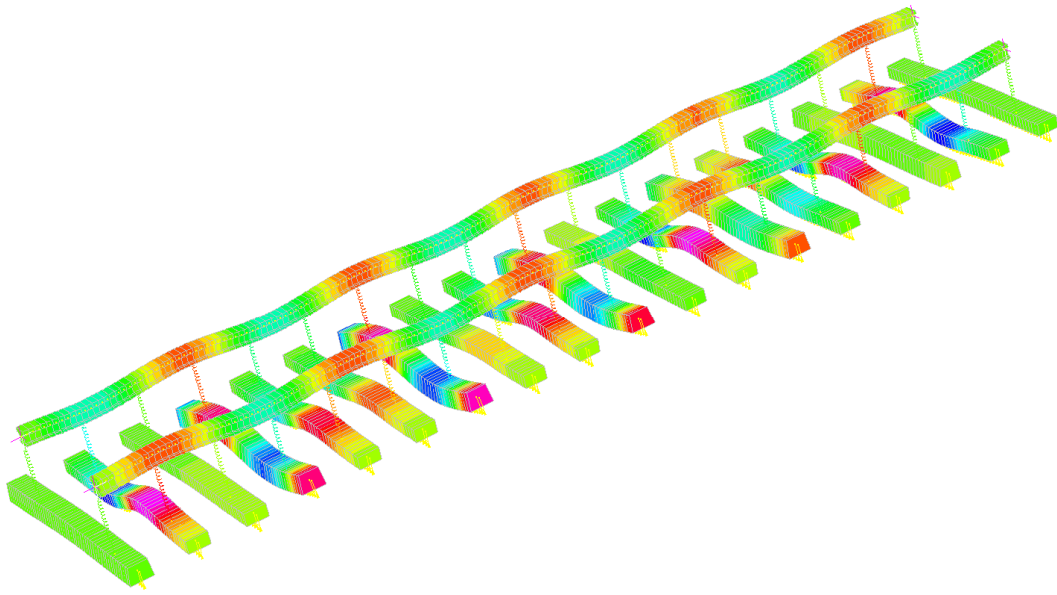


a) out of phase

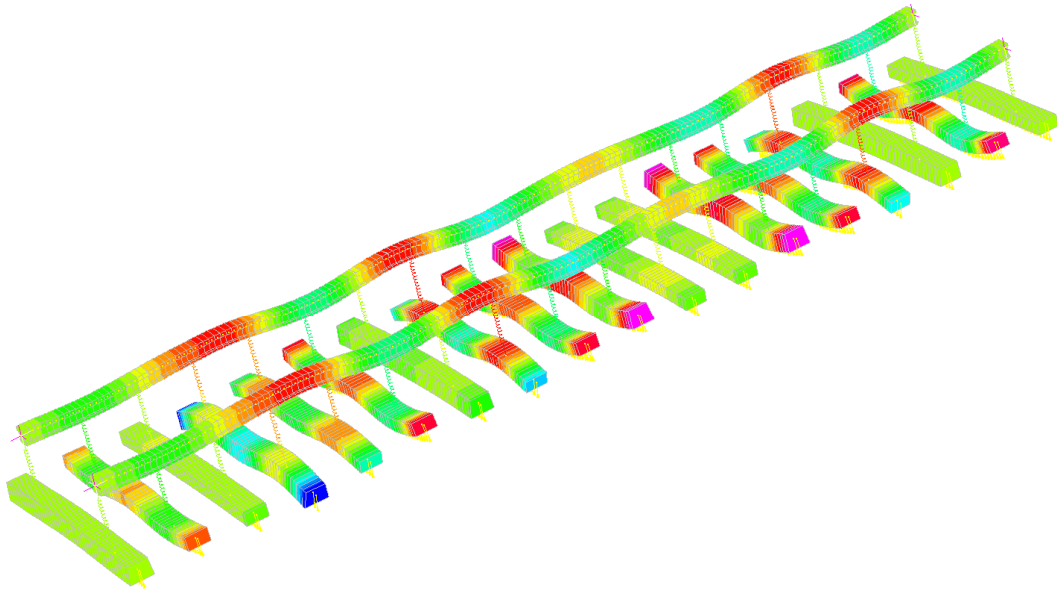


b) out of phase

Fig. 6. Free vibrations of 1:3 interspersed railway tracks (300 to 500 Hz)

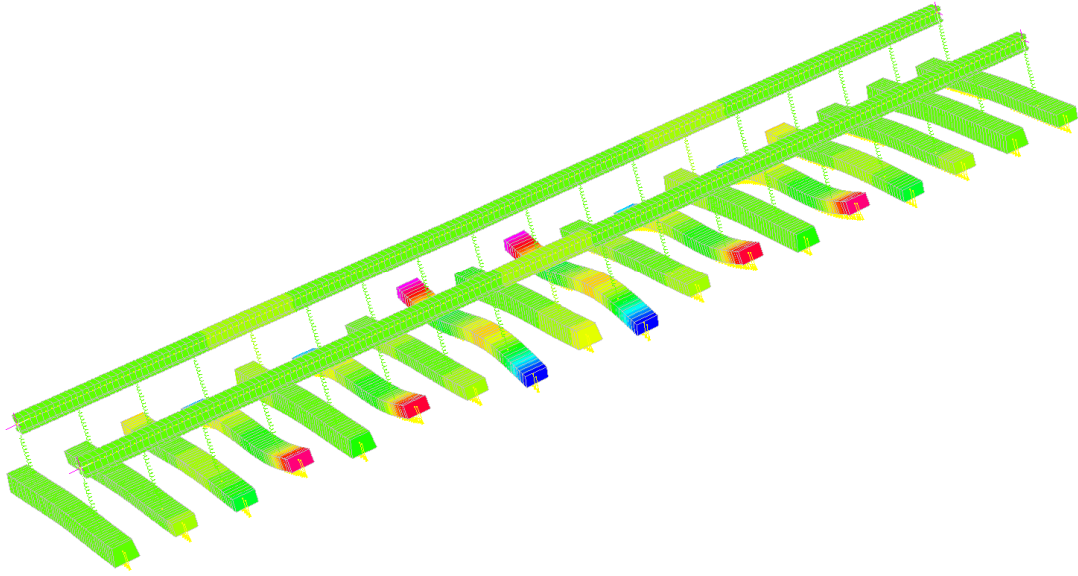


c) out of phase

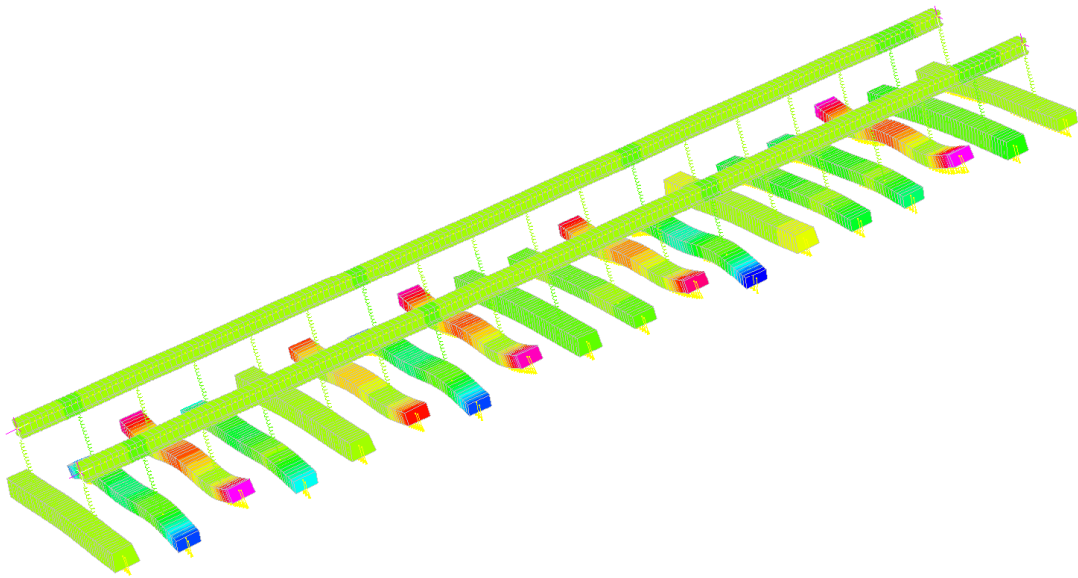


d) out of phase

Fig. 6. Free vibrations of 1:3 interspersed railway tracks (300 to 500 Hz)

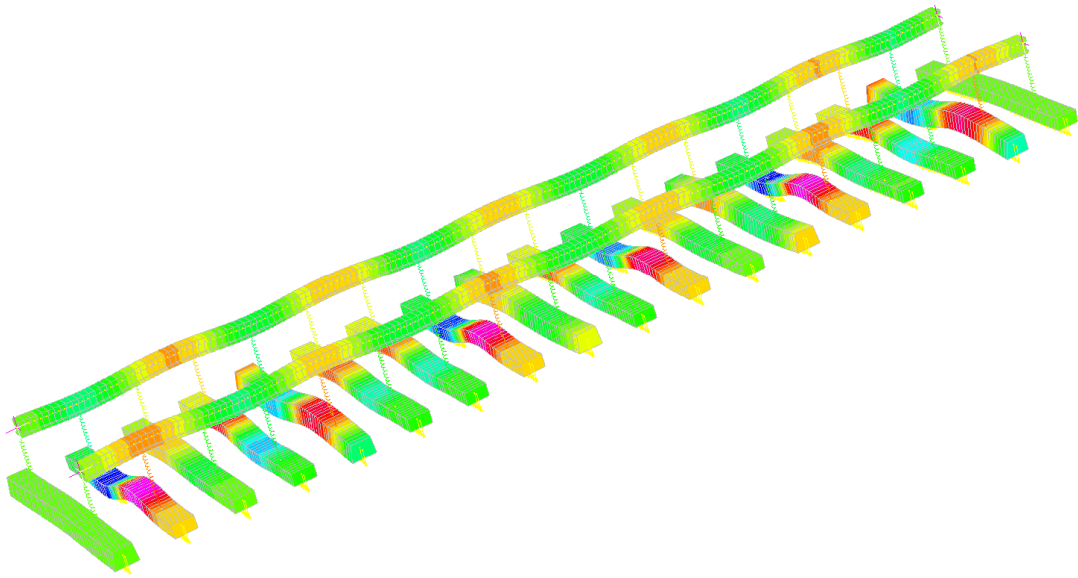


a) out of phase

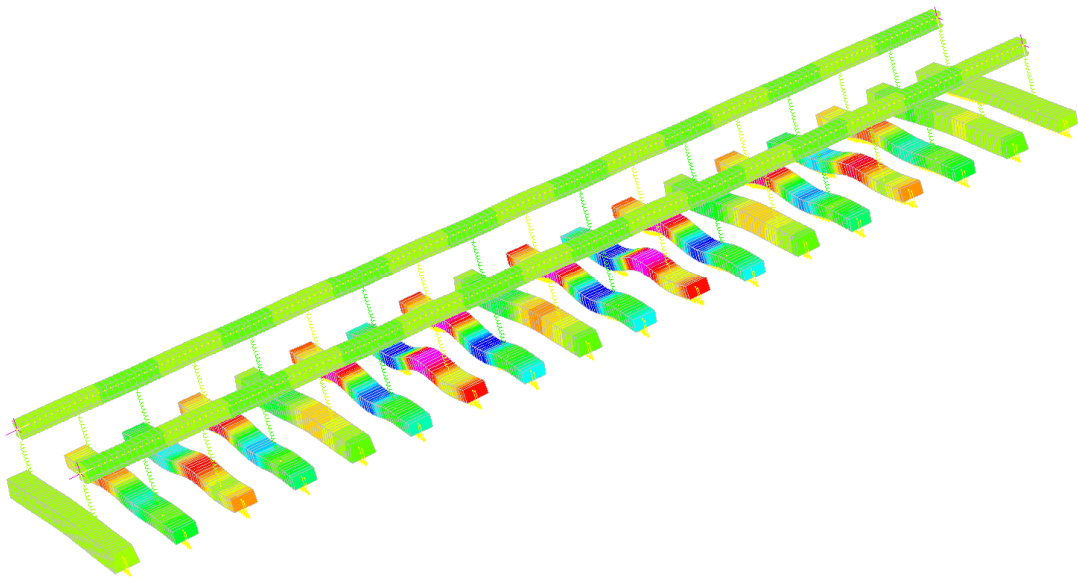


b) out of phase

Fig. 7. Free vibrations of 1:4 interspersed railway tracks (300 to 500 Hz)

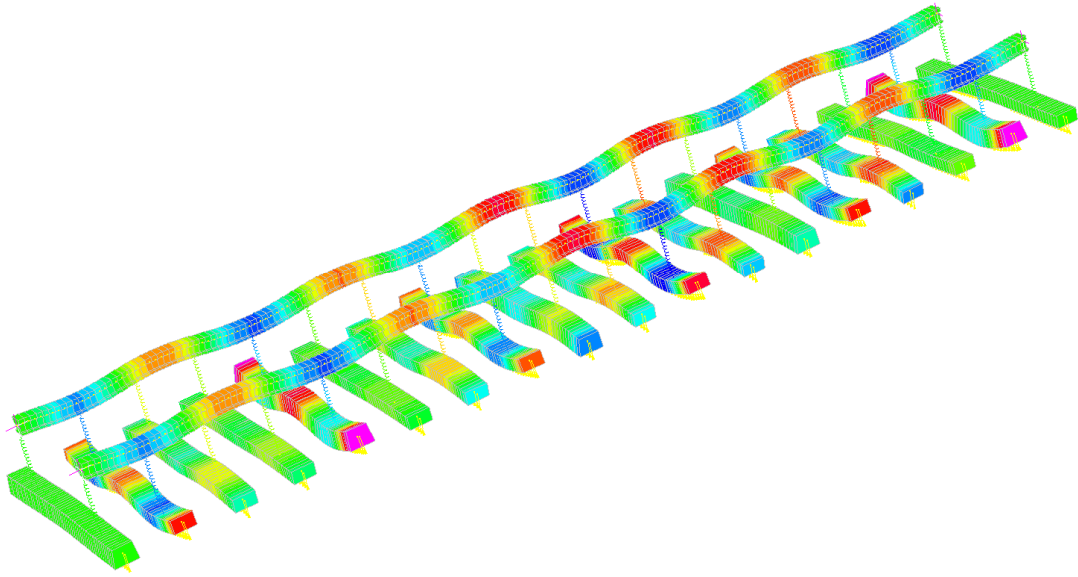


c) out of phase



d) out of phase + rigid body motion

Fig. 7. Free vibrations of 1:4 interspersed railway tracks (300 to 500 Hz)



e) out of phase + rigid body motion

Fig. 7. Free vibrations of 1:4 interspersed railway tracks (300 to 500 Hz)